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Conformal radius: At the interface of traditions

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Abstract

© 2017, Pleiades Publishing, Ltd.H. Behnke's and E. Peschl's definition of plänarkonvexitat leads to the Epstein-type inequalities when applies to the Hartogs domains in C2. One-parameter series of such inequalities reveals the following rigidity phenomenon: the set of the parameters with contensive inequalities is exactly the segment which center corresponds to the well-known Nehari ball. The latter plays the crucial role in the forming the Gakhov class of all holomorphic and locally univalent functions in the unit disk with no more than one-pointed null-sets of the gradients of their conformal radii. The sufficient condition for the piercing of the Nehari sphere out of the Gakhov class is found. We deduce such a condition along the lines of the subordination approach to the proof of Haegi's theorem about the inclusion of any convex holomorphic function into the Gakhov class.

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Keywords

Conformal radius, Epstein inequality, Gakhov class, Gakhov equation, Hartogs domain, hyperbolic derivative, linear convexity, linear-invariant family

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